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Holes in the Probability Density of Strongly Colored Noise Driven Systems[1]

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A qualitative change in the topology of the joint probability density $P(\varepsilon, x)$, which occurs for strongly colored noise in multistable systems, has recently been observed first by analog simulation (F. Moss and F. Marchesoni, *Phys. Lett. A* **131**:322 (1988)) and confirmed by matrix continued fraction methods (Th. Leiber and H. Riskin, unpublished), and by analytic theory (P. Hänggi, P. Jung, and F. Marchesoni, *J. Stat. Phys.*, this issue). Systems studied were of the class $x = -\partial U(x)/\partial x + \varepsilon(t, \tau)$, where $U(x)$ is a multistable potential and $\varepsilon(t, \tau)$ is a colored, Gaussian noise of intensity D, for which $\langle \varepsilon \rangle = 0$, and $\langle \varepsilon(t)\, \varepsilon(s) \rangle = (D/\tau) \exp(-|t-s|/\tau)$. When the noise correlation time τ is smaller than some critical value τ_0, which depends on D, the two-dimensional density $P(\varepsilon, x)$ has the usual topology [P. Jung and H. Risken, *Z. Phys. B* **61**:367 (1985); F. Moss and P. V. E. McClintock, *Z. Phys. B* **61**:381 (1985)]: a pair of local maxima of $P(\varepsilon, x)$, which correspond to a pair of adjacent local minima of $U(x)$, are connected by a single saddle point which lies on the x axis. When $\tau > \tau_0$, however, *the single saddle disappears and is replaced by a pair of off-axis saddles.* A depression, or hole, which is bounded by the saddles and the local maxima thus appears. The most probable trajectory connecting the two potential wells therefore does not pass through the origin for $\tau > \tau_0$, but instead must detour around the local barrier. This observation implies that successful mean-first-passage-time theories of strongly colored noise driven systems must necessarily be two dimensional (Hänggi *et al.*). We have observed these holes for several potentials $U(x)$: (1) a "soft," bistable potential by analog simulation (Moss and Marchesoni); (2) a periodic potential [Th. Leiber, F. Marchesoni, and H. Risken, *Phys. Rev. Lett.* **59**:1381 (1987)] by matrix continued fractions; (3) the usual "hard," bistable potential, $U(x) = -ax^2/2 + bx^4/4$, by analog simulations only; and (4) a random potential for which the forcing $f(x) = -\partial U(x)/\partial x$ is an approximate Gaussian with nonzero correlation length, i.e.,

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colored spatiotemporal noise, by analog simulation. There is a critical curve $\tau_0(D)$ in the τ versus D plane which divides the two topological behaviors. For a fixed value of D, this curve is shifted toward larger values of τ_0 for progressively weaker barriers between the wells. Therefore, strong barriers favor the observation of this topological transformation at smaller values of τ. Recently, an analytic expression for the critical curve, valid asymptotically in the small-D limit, has been obtained (Hänggi *et al.*).

KEY WORDS: Topology of probability density; colored noise; noise-induced topologies; analog simulation; matrix continued fractions; bistable potential; bistability; random potential; noise-correlation-time-induced transition; critical transition; spatiotemporal noise.